**Mean Field Approximation**

So in previous files we saw that our GF expansion was still only presuming, implicitly, weak V0. And as such, the model wasn’t able to predict any magnetic behavior. So now we’ll look use a different approximation: MFT. This won’t place any restrictions on potential strength, and we’d like to see if magnetic behavior could emerge, though, I wonder if we’d lose any chance of describing spin wave excitations, as when go to MFT with the Exchange model (next folder), we also lose spin waves (because replacing two-particle interaction with single particle interaction). So now we’ll employ mean field theory to solve the Hubbard model. Should note that, like with the use of MFT for the Exchange Interaction (next folder), MFT can be unreliable in low dimensions. MFT often works qualitatively well, near the critical point, for dimensions greater than the lower critical dimension (often d = 2) and perfectly well for dimensions greater than the upper critical dimension (often d = 4 or something). But they are unreliable for d < lower critical dimension. I think d = 2 is the lower critical dimension for the Hubbard model, and also for the Exchange model. Can use Renormalization Group methods to investigate when MFT is appropriate, and what to do when it’s not. But anyway, we’ll go back to:



Then the approach is to make H quadratic by expressing the number operators as a mean (a mean in the thermal sense, I believe, and once the energy levels of our H are determined, we can go back and work out this mean self-consistently) + deviation from the mean,



where we presume the average value of njσ doesn’t depend on site, and equals total number of spin σ particles divided by number of lattice sites, Nσ/N = σ. We presume that the deviation is small. We don’t know what these numbers are of course, but we presume to be able to work them out self-consistently. Next we put it into the cuartic part of H, keeping only terms to first order,



Now we’ll fill in what δnjσ is, to get everything back in terms of the familiar number operator.



Seems we can combine the two guys in the second term by switching indices ℓ → j, σ´ → σ in the last term in the bracket [can do this because is symmetric in j, ℓ. I think we can ignore the last two terms as they’re just constants. Well the penultimate one is independent of magnetization because Σσσ = 0, so we should be able to drop it. But maybe we better keep the last term because it’s magnetization dependent?



where 0 = N0/N is the average occupancy of each site regardless of spin. I guess I can define the offsite potential term as:



and then we have, explicitly writing out the creation/annihilation operators:



Now this is technically diagonalizable of course. I guess the standard procedure is to go to Fourier space,



Then,



Let’s define



Then we have:



Now define, as usual,



and then we can say,



Now I don’t want to bother with trying to diagonalize it with eff, so let’s just specialize to the pure Hubbard model for simplicity,



So then we have ‘vacuum’ energy, and excitations above it given by:



and if we’re dealing with a cubic lattice, and δ extends to nearest neighbors only, say, then approximately,



where we use fact that t itself is usually negative. We can put the spectra in terms of the magnetization and overall average occupation number via:



Doing so we have:



Can see there is a competition between the typical band energy and the onsite repulsion. The former, due to Pauli-Exclusion principle (and roughly parabolic spectrum) will make us want to minimize M, but the latter to maximize it (in positive or negative direction).